The *Zhang lattice*: a simple lattice naturally has type-II Dirac points

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I review the discovery as well as the band structure of the *Zhang lattice*.

Dirac materials [1] are special since there are Dirac points in their corresponding band structures, which means that the excited quasi-particle is fermionic and obeys the massless Dirac equation [2]. Probably, the most frquently used Dirac materials are the graphene (or the honeycomb lattice), the Lieb lattice and the kagome lattice [3]. However, the Dirac point supported by these lattices are type-I [4]. Considering half-filling, the Fermi surface is a point for the type-I Dirac point, a line for the type-III Dirac point and a pair of crossing lines for the type-II Dirac point. In comparison with the materials that support type-I Dirac point, materials that support the type-II and the type-III Dirac points are much more rare, especially those support the type-II Dirac point. To obtain the type-II Dirac point, various methods are developed [5–10]. I would like to state that all the efforts are not direct. One cannot help wondering that is there a kind of material or artificial material that naturally possesses the type-II Dirac point, just like a graphene possessing the type-I Dirac point? To this end, I started pondering on this question since I noticed Ref. [6], and finally succeeded in 2020 [11] after many attempts and efforts [12–14].

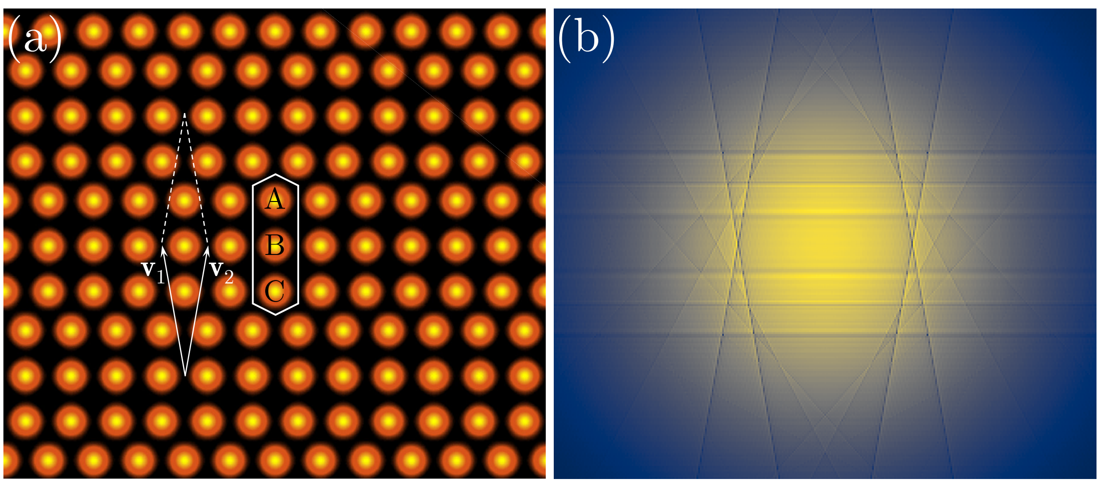

FIG. 1. (a) Landscape of the *Zhang lattice*. In each unit cell (indicated by a hexagon) there are three sites labeled as A, B and C. The basis vectors of the Bravais lattice are $\mathbf{v}_1 = [-a/2, 2a + \sqrt{3}a/2]$ and $\mathbf{v}_2 = [+a/2, 2a + \sqrt{3}a/2]$ with a being the lattice constant. (b) Far-field diffraction pattern with the innermost hexagon being the first Brillouin zone.

The designed lattice, i.e., the *Zhang lattice*, is shown in Fig. 1(a), which has three sites in one unit cell. Its corresponding far-field diffraction pattern in Fig. 1(b) shows the first Brillouin zone (the innermost hexagon) clearly. In Ref. [11], the six corners of the first Brillouin zone are exhibited:

$$\left(\pm\frac{(20+8\sqrt{3})\pi}{(19+8\sqrt{3})a}, 0\right) \quad \text{and} \quad \left(\pm\frac{(18+8\sqrt{3})\pi}{(19+8\sqrt{3})a}, \pm\frac{2\pi}{(4+\sqrt{3})a}\right). \tag{1}$$

Therefore, I believe the complexity of the *Zhang lattice* is in the same level as that of the Lieb lattice and the kagome lattice, and the *Zhang lattice* is a simple lattice.

The band structure of the *Zhang lattice* was calculated based on the both discrete model (i.e., the tight-binding method) and the continuous model (i.e., the Schrödinger-like paraxial wave equation), and both results demonstrate

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the existence of the type-II Dirac points. According to the tight-binding method with merely nearest-neighbor hopping being considered, the Hamiltonian can be written as

$$\mathcal{H}=t\begin{bmatrix} 2\cos[\mathbf{k}\cdot(\mathbf{v}_2-\mathbf{v}_1)] & 1 & \exp(-i\mathbf{k}\cdot\mathbf{v}_1)+\exp(-i\mathbf{k}\cdot\mathbf{v}_2) \\ 1 & 2\cos[\mathbf{k}\cdot(\mathbf{v}_2-\mathbf{v}_1)] & 1 \\ \exp(+i\mathbf{k}\cdot\mathbf{v}_1)+\exp(+i\mathbf{k}\cdot\mathbf{v}_2) & 1 & 2\cos[\mathbf{k}\cdot(\mathbf{v}_2-\mathbf{v}_1)] \end{bmatrix}, \tag{2}$$

with $\mathbf{k}=[k_x,k_y]$ being the Bloch momentum and t the hopping strength. The band structure, i.e., the eigenvalues of the Hamiltonian versus k_x and k_y, is shown in Fig. 2(a). There are three bands in the band structure, and the intersections between each two bands are type-II Dirac points. The appearance of the type-II Dirac point is natural and no additional operation is required onto the *Zhang lattice*.

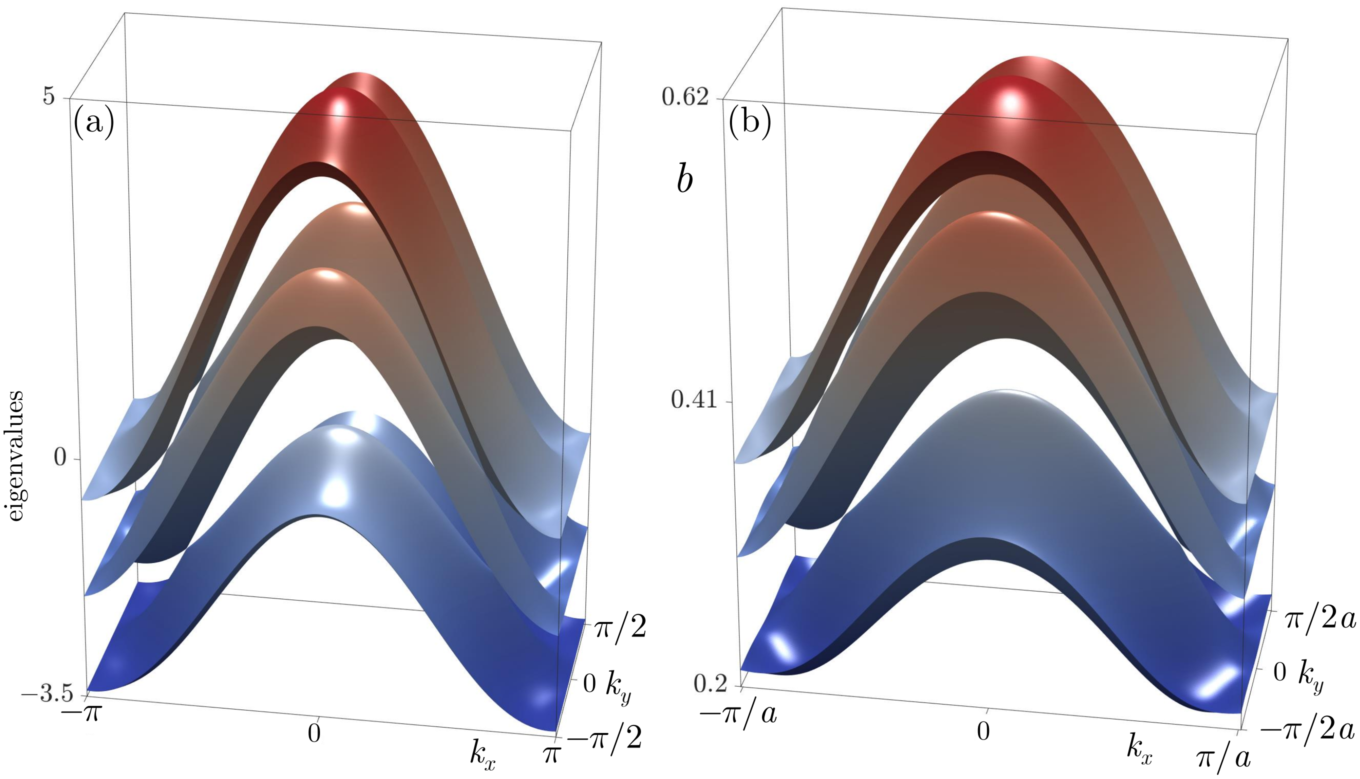

FIG. 2. (a) Band structure of the *Zhang lattice* by assuming $a=1$ and $t=1$ based on the tight-binding method. (b) Band structure of the *Zhang lattice* based on the continuous model with $a=3$ and $p=5$.

The continuous model includes all hoppings as well as the concrete profile of the sites, and therefore is more accurate than the discrete model. Assuming the *Zhang lattice* is inscribed in a transparent optical medium (e.g., the fused silica) by the femto-second laser direct writing technique, and the dimensionless Schrödinger-like paraxial wave equation should be written as

$$i\frac{\partial\psi}{\partial z}=-\frac{1}{2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi-\mathcal{R}(x,\gamma)\psi, \tag{3}$$

where $\mathcal{R}$ is the *Zhang lattice* that can be depicted by Gaussian functions

$$\mathcal{R}(x,y)=p\sum_{m,n}\exp\left(-\frac{(x-x_{m,n})^2}{\sigma_x^2}-\frac{(y-y_{m,n})^2}{\sigma_y^2}\right), \tag{4}$$

with p being the lattice depth, (σ_x,σ_y) charging for the beam width, and $(x_{m,n},y_{m,n})$ being the coordinates of the lattice grids. Considering a set of real experimental parameters [15, 16], e.g., $\sigma_x=0.25$ ($2.5\,\mu$m), $\sigma_y=0.75$ ($7.5\,\mu$m), $a=3$ ($30\,\mu$m), $\lambda=600\,$nm, and $p=5$ (the refractive index change $\sim 5.5\times10^{-4}$), the corresponding band structure of the *Zhang lattice* is shown in Fig. 2(b), by introducing the ansatz $\psi(x,y,z)=u(x,y)\exp(ibz)$ for Eq. (3) with b

being the propagation constant and $u(x, y)$ the Bloch state. The result based on the continuous model is quite similar to that based on the discrete model, and the type-II Dirac points are definitely supported by the *Zhang lattice*.

Note that the *Zhang lattice* has been induced in photorefractive SBN crystals [17], and surely in atomic vapors [18] with the aid of a spatial modulator. In the *Zhang lattice*, the conical diffraction [11], the Klein tunneling [11], the scalar as well as the vector valley Hall edge solitons have been reported [17, 19, 20]. In the future, I believe more and more interesting physical principles and phenomena will be reported and verified based on the *Zhang lattice*.

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